An Introduction to Click Models for Web Search

(Morning block 1)

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SIGIR 2015 Tutorial

Who are we?

Aleksandr

Software engineer

at Google

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Aleksandr
Software engineer
at Google

Ilya
Postdoc at
U. Amsterdam

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Software engineer
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Ilya
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Maarten
Professor at
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Aims

- **Describe existing click models** in a unified way, so that different models can easily be related to each other

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- Summarize current efforts on **click model evaluation** – evaluation approaches, datasets and software packages

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- **Compare** commonly used click models
- Provide **ready-to-use formulas and implementations** of existing click models and detail parameter estimation procedures to facilitate the development of new ones
- Summarize current efforts on **click model evaluation** – evaluation approaches, datasets and software packages
- Provide an overview of click model **applications and directions** for future development of click models

Structure of the tutorial

- Two parts, with two blocks each
  - An Introduction to Click Models for Web Search
  - Advanced Click Models and their applications to IR

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  - An Introduction to Click Models for Web Search
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- Part I (this morning)
  - Introduction
  - Basic click models
  - Inference for click models
  - Break
  - Demo
  - Evaluation
  - Data and tools
  - Results on the basic click models
  - Recap

Materials

- Complete draft of the book on which the tutorial is based:
  - Aleksandr Chuklin, Ilya Markov, Maarten de Rijke. *Click Models for Web Search*. Synthesis Lectures on Information Concepts, Retrieval, and Services. Morgan & Claypool, July, 2015

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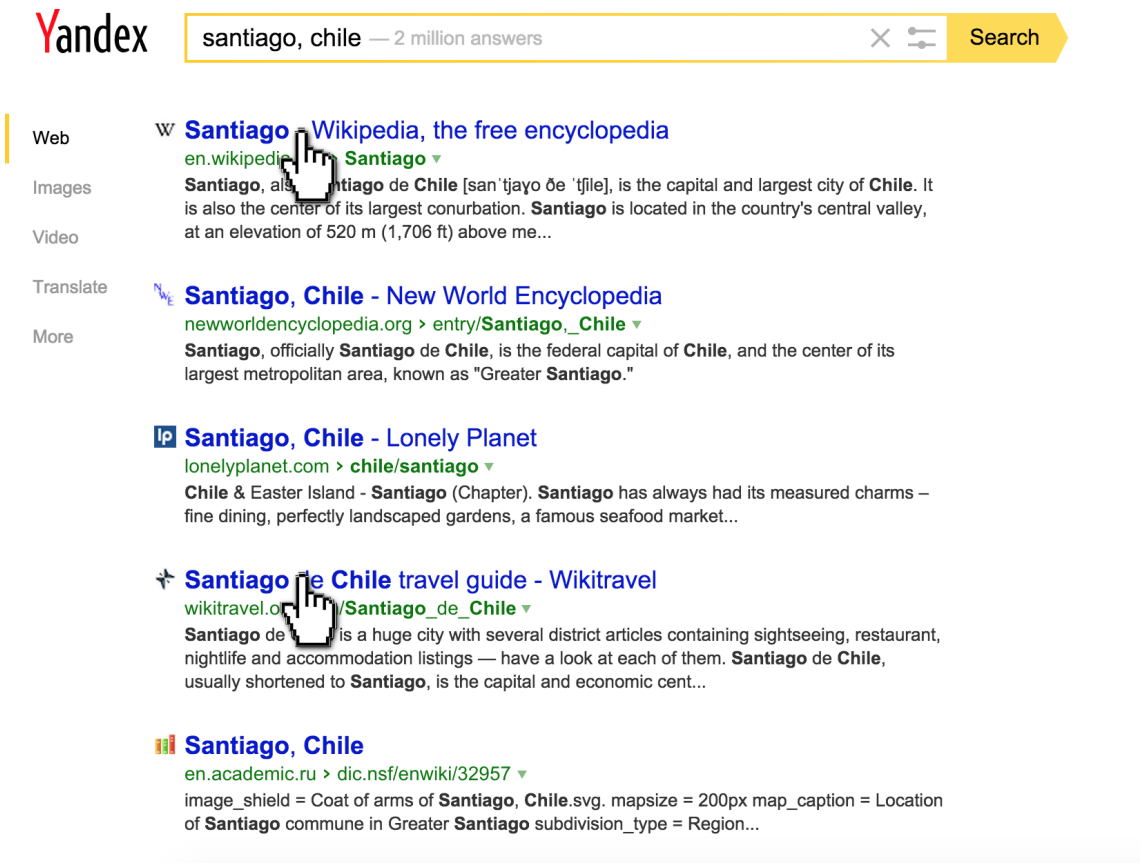
- See `http://clickmodels.weebly.com` for updates and additional materials

Morning block 1 – Outline

What is a click model?



Notation

Expression	Meaning
u	A document
q	A user's query
r	The rank of a document
u_r	A document at rank r
r_u	The rank of a document u

Notation

Expression	Meaning
c	A placeholder for any concept associated with a SERP (e.g., query-document pair, rank, etc.)
$\mathcal{S}$	A set of user search sessions
$\mathcal{S}_c$	A set of user search sessions containing a concept c
X_c	An event X applied to a concept c
x_c	The value that a random variable X takes, when applied to a concept c
$x_c^{(s)}$	The value that a random variable X takes, when applied to a concept c in a particular session s

Morning block 1 – Outline

Why click models?

Why click models?

- Understand users

Why click models?

- Understand users
- Simulate users

Why click models?

- Understand users
- Simulate users
- Evaluate search

Why click models?

- Understand users
- Simulate users
- Evaluate search
- Improve search

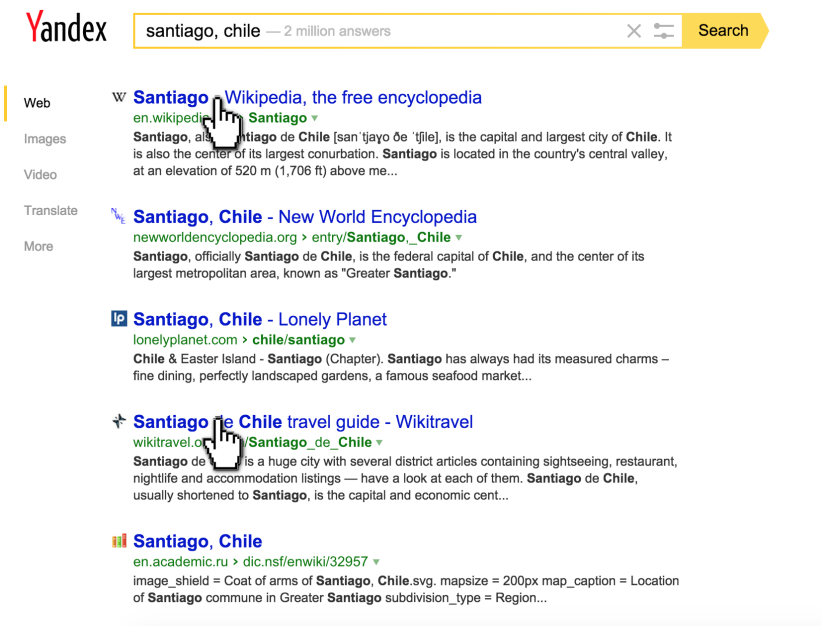
Morning block 1 – Outline

Basic click models

- Random click model
- CTR models
- Position-based model
- Cascade model
- Dependent click model
- Dynamic Bayesian network model
- User browsing model

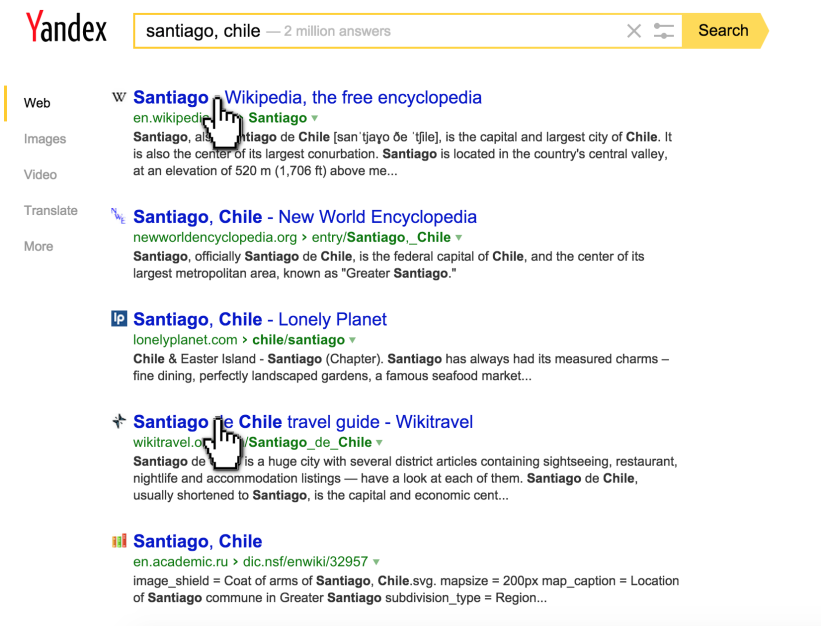
Random click model

?



Random click model

$$P(C_u = 1) = \rho$$



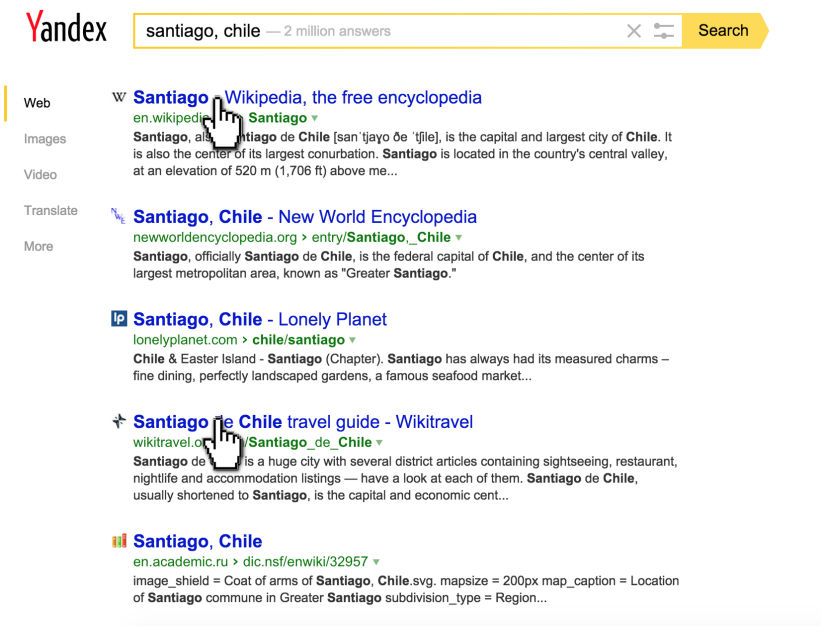
CTR models

Rank-based CTR:

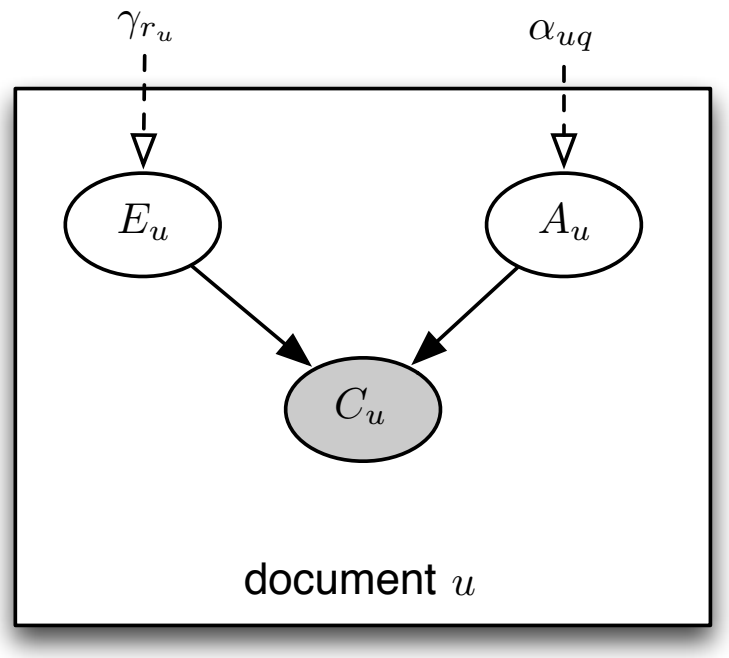
$$P(C_r = 1) = \rho_r$$

Document-based CTR:

$$P(C_u = 1) = \rho_{uq}$$



Position-based model

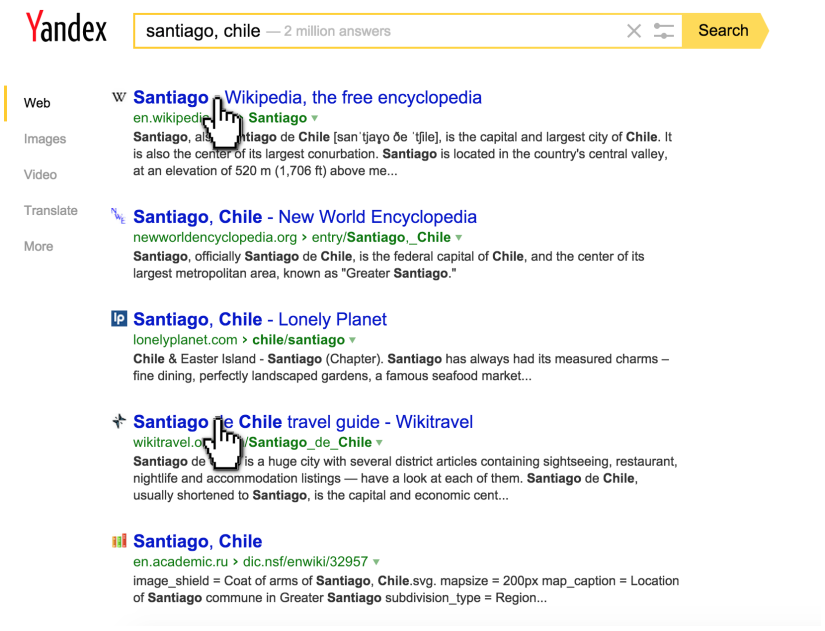


Position-based model

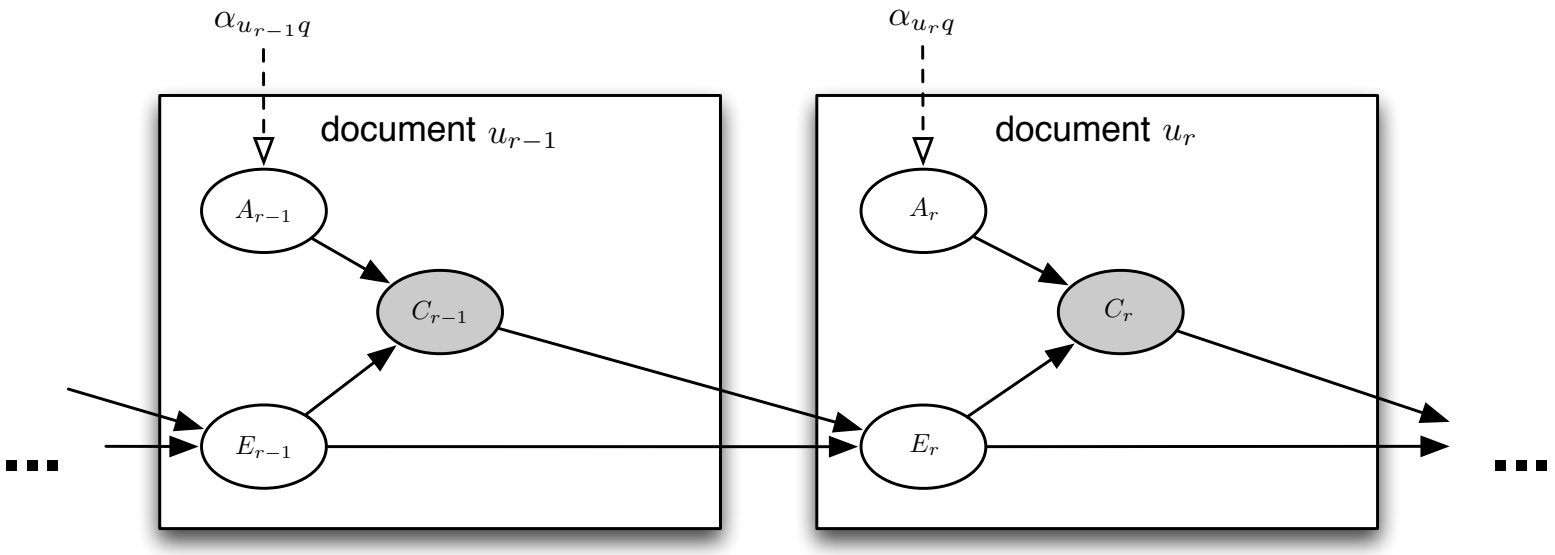
$$P(C_u = 1) = P(E_u = 1) \cdot P(A_u = 1)$$

$$P(A_u = 1) = \alpha_{uq}$$

$$P(E_u = 1) = \gamma_{r_u}$$



Cascade model



Cascade model

$$E_r = 1 \text{ and } A_r = 1 \Leftrightarrow C_r = 1$$

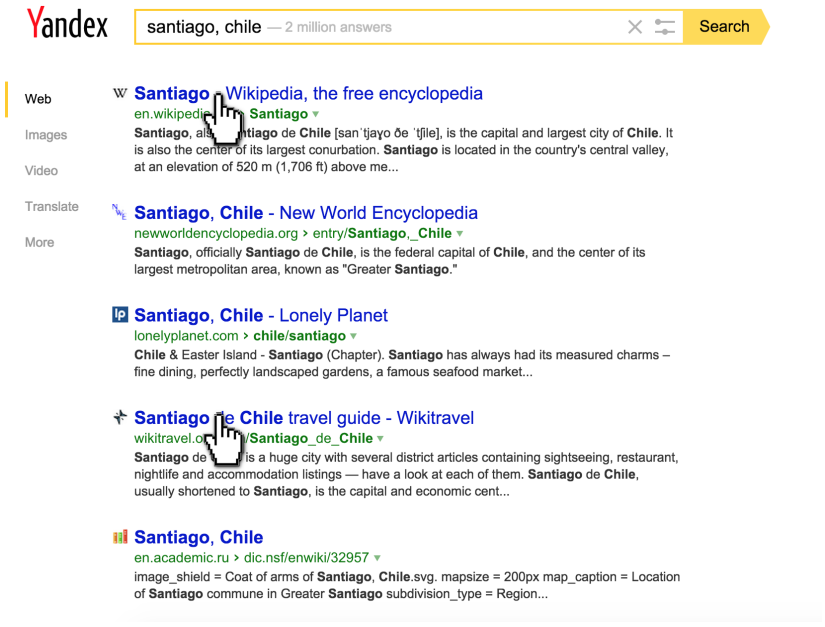
$$P(A_r = 1) = \alpha_{u_r q}$$

$$P(E_1 = 1) = 1$$

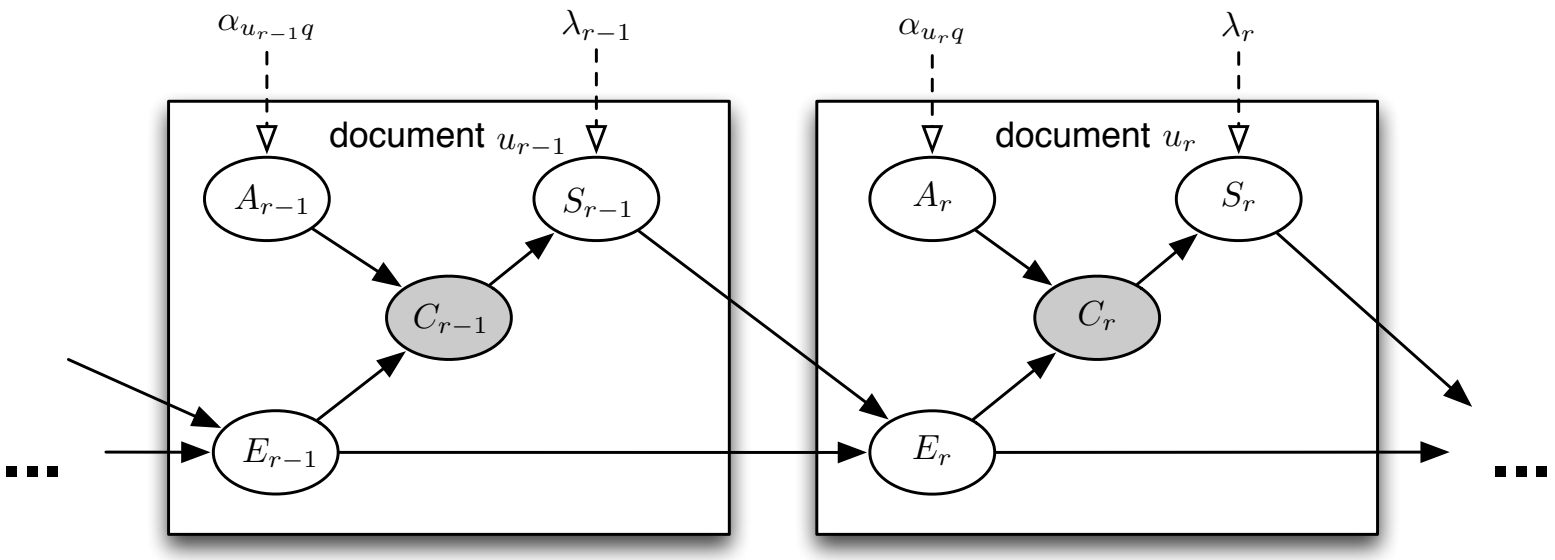
$$P(E_r = 1 \mid E_{r-1} = 0) = 0$$

$$P(E_r = 1 \mid C_{r-1} = 1) = 0$$

$$P(E_r = 1 \mid E_{r-1} = 1, C_{r-1} = 0) = 1$$



Dependent click model



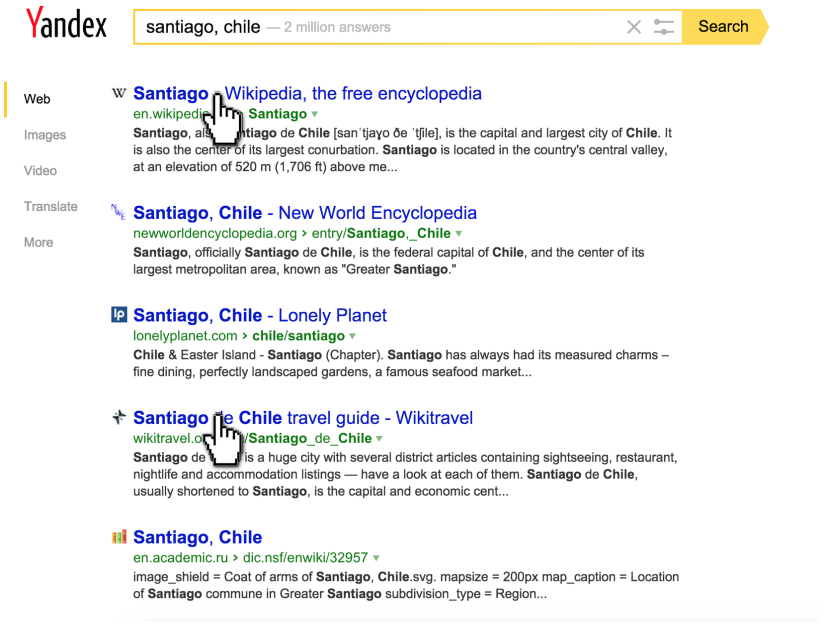
Dependent click model

$$P(E_r = 1 \mid S_{r-1} = 1) = 0$$

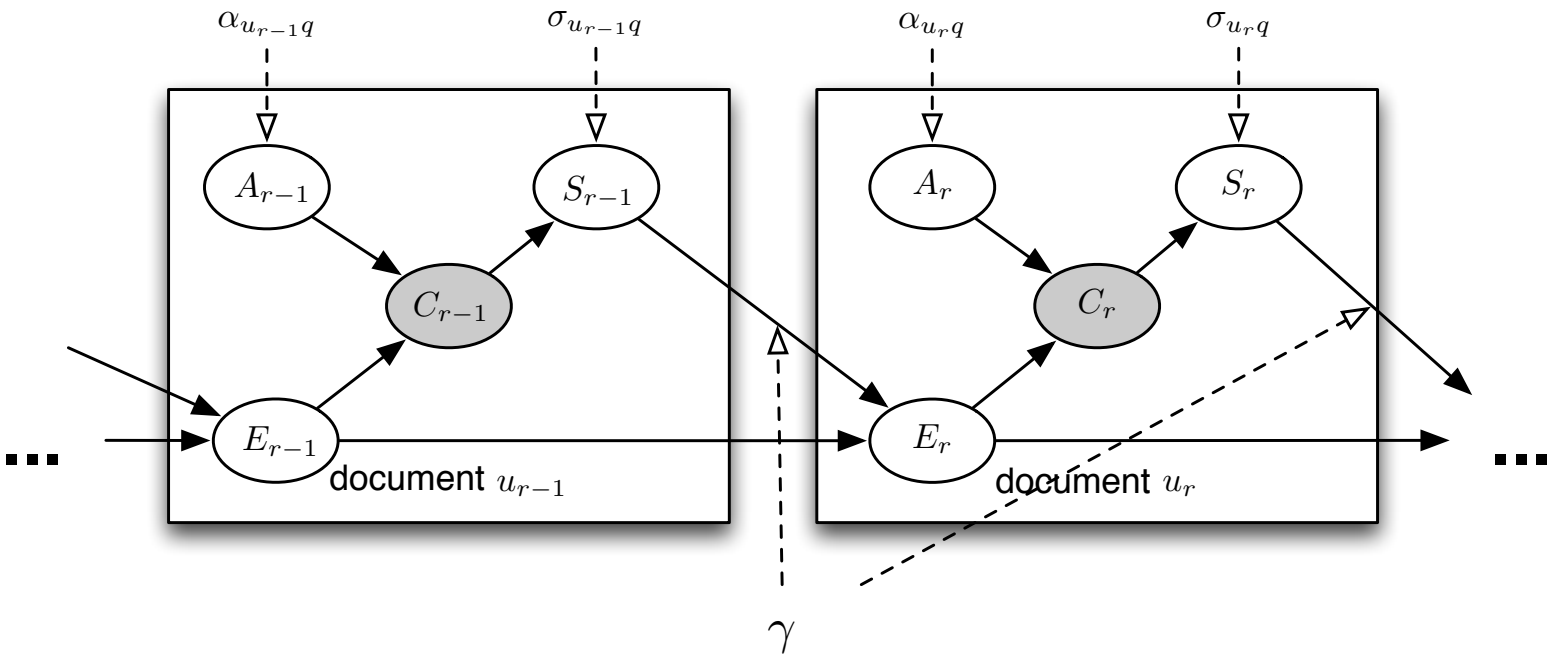
$$P(E_r = 1 \mid E_{r-1} = 1, S_{r-1} = 0) = 1$$

$$P(S_r = 1 \mid C_r = 0) = 0$$

$$P(S_r = 1 \mid C_r = 1) = 1 - \lambda_r$$



Dynamic Bayesian network model

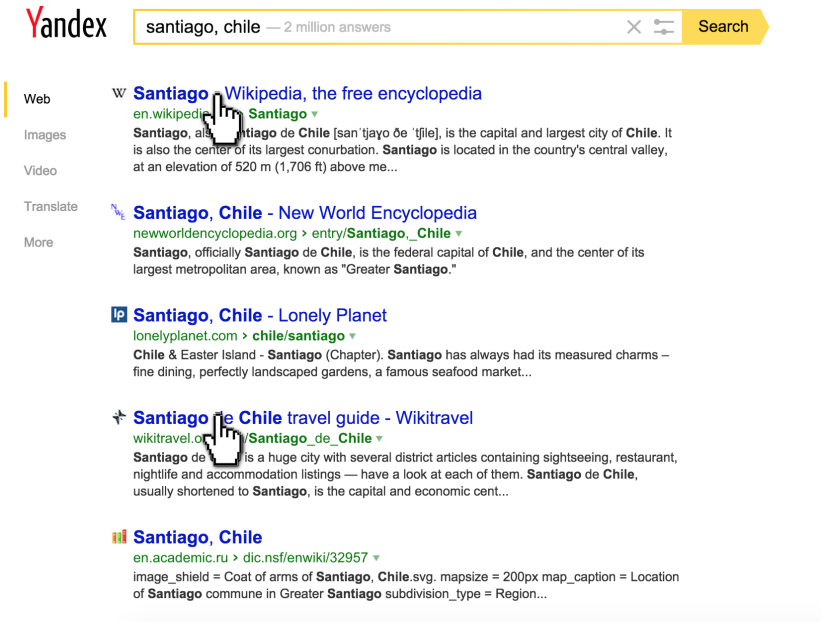


Dynamic Bayesian network model

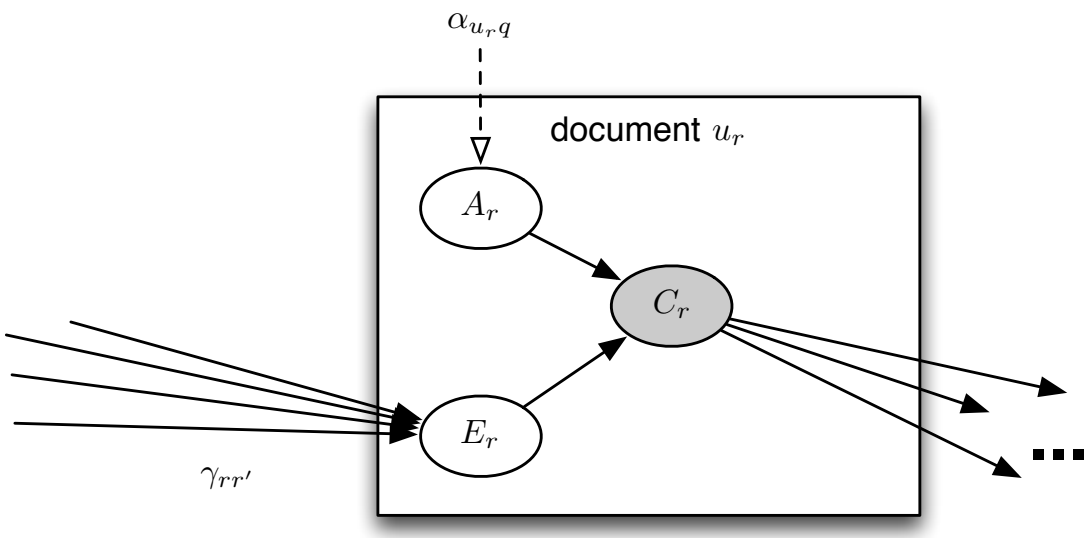
$$P(E_r = 1 \mid S_{r-1} = 1) = 0$$

$$P(E_r = 1 \mid E_{r-1} = 1, S_{r-1} = 0) = \gamma$$

$$P(S_r = 1 \mid C_r = 1) = \sigma_{u_r q}$$

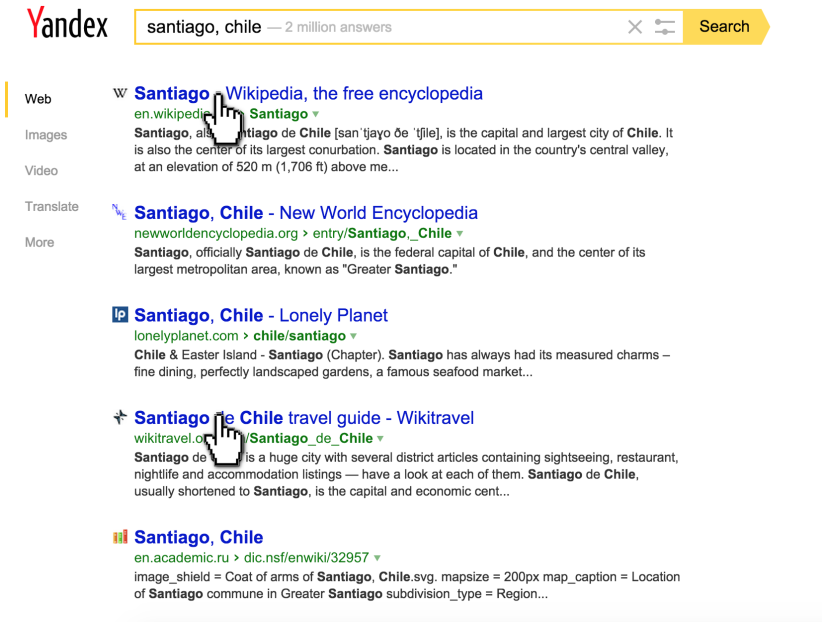


User browsing model



User browsing model

$$P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'}$$



Basic click models summary

Model	$P(A_u = 1)$	$P(S_u = 1 \mid C_u = 1)$	$P(E_u = 1 \mid \mathbf{E}_{<r_u}, \mathbf{S}_{<r_u}, \mathbf{C}_{<r_u})$
RCM	constant (ρ)	N/A	constant (1)
RCTR	constant (1)	N/A	rank (ρ_r)
DCTR	query-document (ρ_{uq})	N/A	constant (1)
PBM	query-doc (α_{uq})	N/A	rank (γ_r)
CM	query-doc (α_{uq})	constant (1)	constant (1 or 0)
DCM	query-doc (α_{uq})	rank ($1 - \lambda_r$)	constant (1 or 0)
DBN	query-doc (α_{uq})	query-doc (σ_{uq})	constant (γ)
UBM	query-doc (α_{uq})	N/A	other ($\gamma_{rr'}$)

Morning block 1 – Outline

Click probabilities

Click probabilities

- Full probability:

$$P(C_r = 1)$$

Click probabilities

- Full probability:

$$P(C_r = 1)$$

- Conditional probability:

$$P(C_r = 1 \mid C_1, \ldots C_{r-1})$$

Full click probability

$$P(C_u = 1) = P(C_u = 1 \mid E_{r_u} = 1) \cdot P(E_{r_u} = 1) = \alpha_{uq}\epsilon_{r_u}$$

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$$\epsilon_{r+1} = P(E_{r+1} = 1)$$

Full click probability

$$P(C_u = 1) = P(C_u = 1 \mid E_{r_u} = 1) \cdot P(E_{r_u} = 1) = \alpha_{uq}\epsilon_{r_u}$$

$$\begin{aligned}\epsilon_{r+1} &= P(E_{r+1} = 1) \\ &= P(E_r = 1) \cdot P(E_{r+1} = 1 \mid E_r = 1)\end{aligned}$$

Full click probability

$$P(C_u = 1) = P(C_u = 1 \mid E_{r_u} = 1) \cdot P(E_{r_u} = 1) = \alpha_{uq}\epsilon_{r_u}$$

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\epsilon_{r+1} &= P(E_{r+1} = 1) \\
&= P(E_r = 1) \cdot P(E_{r+1} = 1 \mid E_r = 1) \\
&= \epsilon_r \cdot \big(P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(C_r = 1 \mid E_r = 1) + \\
&\qquad P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(C_r = 0 \mid E_r = 1)\big)
\end{aligned}$$

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&\qquad P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(C_r = 0 \mid E_r = 1)\big)
\end{aligned}$$

DBN: $\epsilon_{r+1} = \epsilon_r \left((1 - \sigma_{uq})\, \gamma\alpha_{uq} + \gamma\,(1 - \alpha_{uq})\right)$

Conditional click probability

$$
\begin{aligned}
P(C_u = 1 \mid \mathbf{C}_{<r_u}) &= P(C_u = 1 \mid E_{r_u} = 1, \mathbf{C}_{<r_u}) \cdot P(E_{r_u} = 1 \mid \mathbf{C}_{<r_u}) \\
&= \alpha_{uq} \epsilon_{r_u}
\end{aligned}
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&= P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(E_r = 1 \mid C_r = 1, \mathbf{C}_{<r}) \cdot c_r^{(s)} + \\
&\quad P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(E_r = 1 \mid C_r = 0, \mathbf{C}_{<r}) \cdot (1 - c_r^{(s)})
\end{aligned}
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&= P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot c_r^{(s)} + \\
&\quad P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot \frac{\epsilon_r(1 - \alpha_{u_r q})}{1 - \alpha_{u_r q}\epsilon_r}(1 - c_r^{(s)})
\end{aligned}$$

Conditional click probability

$$P(C_u = 1 \mid \mathbf{C}_{<r_u}) = P(C_u = 1 \mid E_{r_u} = 1, \mathbf{C}_{<r_u}) \cdot P(E_{r_u} = 1 \mid \mathbf{C}_{<r_u})$$
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&\quad P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot \frac{\epsilon_r(1 - \alpha_{u_r q})}{1 - \alpha_{u_r q}\epsilon_r}(1 - c_r^{(s)})
\end{aligned}$$

DCM: $\epsilon_{r+1} = \lambda_r c_r^{(s)} + \frac{(1 - \alpha_{u_r q})\epsilon_r}{1 - \alpha_{u_r q}\epsilon_r}(1 - c_r^{(s)})$

Morning block 1 – Outline

Click model

Click model

- Set of events/random variables
- Set of dependencies between these events
- Correspondence between the model's parameters and features of a query and results

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Parameter estimation

- Maximum likelihood estimation
- Expectation maximization
- Alternative estimation methods

MLE for random click model

$$P(C_u = 1) = \rho$$

MLE for random click model

$$P(C_u = 1) = \rho$$

$$\mathcal{L} = \prod_{s \in \mathcal{S}} \prod_{u \in s} \rho^{c_u^{(s)}} (1 - \rho)^{1 - c_u^{(s)}}$$

MLE for random click model

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$$\mathcal{LL} = \sum_{s \in \mathcal{S}} \sum_{u \in s} \left(c_u^{(s)} \log(\rho) + (1 - c_u^{(s)}) \log(1-\rho) \right)$$

MLE for random click model

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$$\mathcal{LL} = \sum_{s \in \mathcal{S}} \sum_{u \in s} \left(c_u^{(s)} \log(\rho) + (1 - c_u^{(s)}) \log(1-\rho) \right)$$

$$\rho = \frac{\sum_{s \in \mathcal{S}} \sum_{u \in s} c_u^{(s)}}{\sum_{s \in \mathcal{S}} |s|}$$

MLE for dependent click model

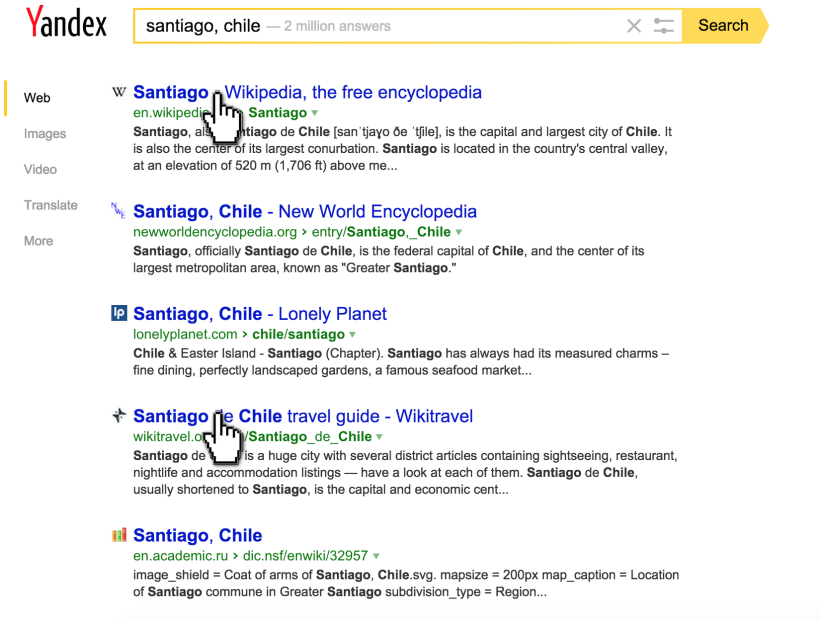
$$P(A_u = 1) = \alpha_{uq}$$

$$P(E_r = 1 \mid E_{r-1} = 1, S_{r-1}) = 1 - S_{r-1}$$

$$P(S_r = 1 \mid C_r = 0) = 0$$

$$P(S_r = 1 \mid C_r = 1) = 1 - \lambda_r$$

$$C_u = 1 \Leftrightarrow E_{r_u} = 1, A_u = 1$$



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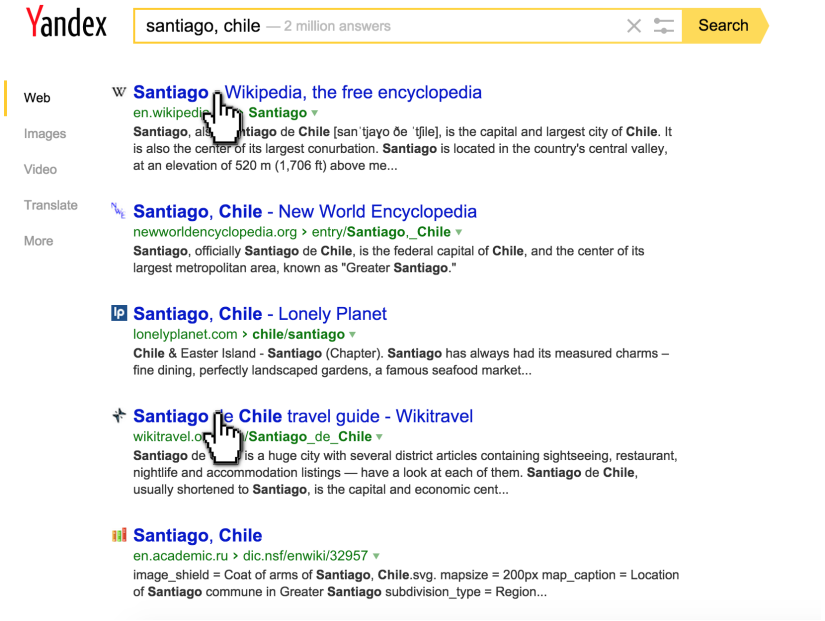

$$S_r = 1 \iff r = l,$$

where l is the last-clicked rank

MLE for dependent click model: Attractiveness

$$P(A_u = 1) = \alpha_{uq}$$

$$\forall r \leq l: \; A_{u_r} = 1 \iff C_{u_r} = 1$$

MLE for dependent click model: Attractiveness

$$P(A_u = 1) = \alpha_{uq}$$

$$\forall r \leq l: \; A_{u_r} = 1 \iff C_{u_r} = 1$$

$$\mathcal{L}(\alpha_{uq}) = \prod_{s \in \mathcal{S}_{uq}} \alpha_{uq}^{\mathcal{I}(A_u^{(s)}=1)} (1 - \alpha_{uq})^{1-\mathcal{I}(A_u^{(s)}=1)}$$

$$\mathcal{LL}(\alpha_{uq}) = \sum_{s \in \mathcal{S}_{uq}} \left(\mathcal{I}(\dots) \log(\alpha_{uq}) + (1 - \mathcal{I}(\dots)) \log(1 - \alpha_{uq})\right)$$

MLE for dependent click model: Attractiveness

$$P(A_u = 1) = \alpha_{uq}$$

$$\forall r \leq l: \; A_{u_r} = 1 \iff C_{u_r} = 1$$

$$\mathcal{L}(\alpha_{uq}) = \prod_{s \in \mathcal{S}_{uq}} \alpha_{uq}^{\mathcal{I}(A_u^{(s)}=1)} (1 - \alpha_{uq})^{1-\mathcal{I}(A_u^{(s)}=1)}$$

$$\mathcal{LL}(\alpha_{uq}) = \sum_{s \in \mathcal{S}_{uq}} \left(\mathcal{I}(\dots) \log(\alpha_{uq}) + (1 - \mathcal{I}(\dots)) \log(1 - \alpha_{uq})\right)$$

$$\alpha_{uq} = \frac{\sum_{s \in \mathcal{S}_{uq}} \mathcal{I}(A_u^{(s)} = 1)}{|\mathcal{S}_{uq}|} = \frac{\sum_{s \in \mathcal{S}_{uq}} \mathcal{I}(C_u^{(s)} = 1)}{|\mathcal{S}_{uq}|} = \frac{\sum_{s \in \mathcal{S}_{uq}} c_u^{(s)}}{|\mathcal{S}_{uq}|}$$

MLE for dependent click model: Continuation

$$P(S_r = 0) = \lambda_r$$

$$S_r = 1 \iff C_r = 1, r = l$$

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MLE for dependent click model

$$\alpha_{uq} = \frac{1}{|\mathcal{S}_{uq}|} \sum_{s \in \mathcal{S}_{uq}} c_u^{(s)}$$

$$\lambda_r = \frac{1}{|\mathcal{S}_r|} \sum_{s \in \mathcal{S}_r} \mathcal{I}(r \neq l)$$

Expectation maximization

$$\mathcal{LL} = \sum_{s \in \mathcal{S}} \log \left(\sum_{\mathbf{X}} P\left(\mathbf{X}, \mathbf{C}^{(s)} \mid \boldsymbol{\Psi}\right) \right)$$

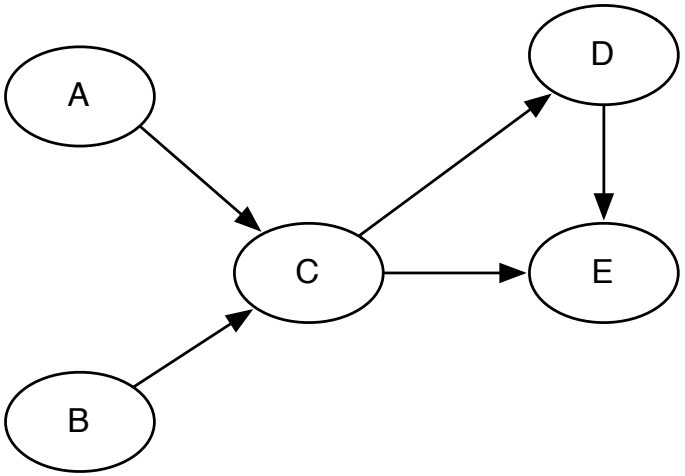
Expectation maximization

$$\mathcal{LL} = \sum_{s \in \mathcal{S}} \log \left(\sum_{\mathbf{X}} P\left(\mathbf{X}, \mathbf{C}^{(s)} \mid \boldsymbol{\Psi}\right) \right)$$

$$Q = \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}} \left[\log P\left(\mathbf{X}, \mathbf{C}^{(s)} \mid \boldsymbol{\Psi}\right) \right]$$

Expectation maximization: E-step (grouping)

Each node X depends only on its parents $\mathcal{P}(X)$



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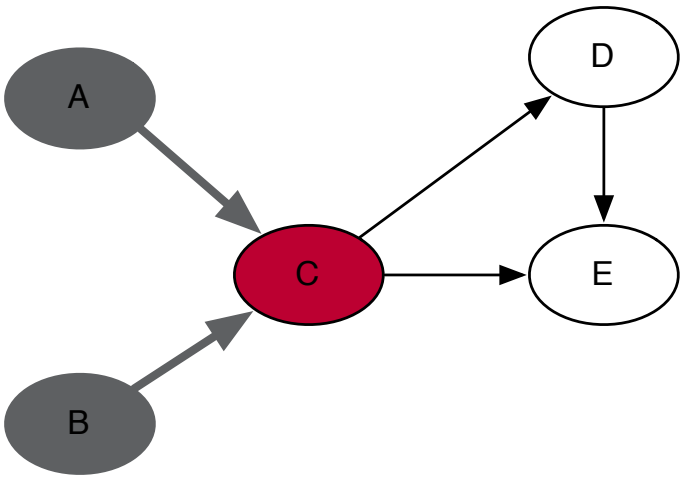
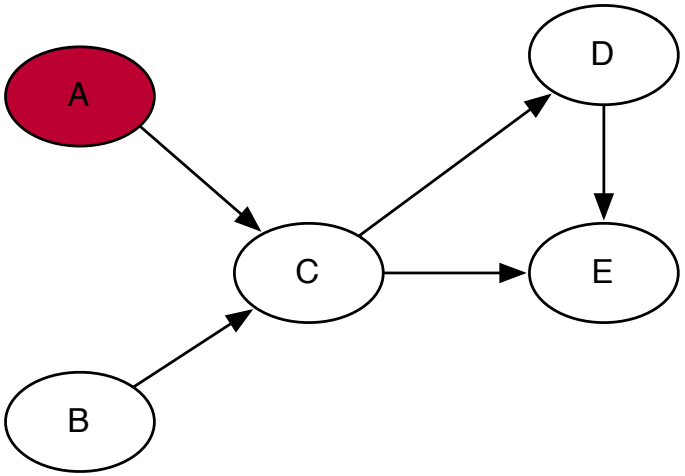


Expectation maximization: E-step (grouping)

Each node X depends only on its parents $\mathcal{P}(X)$

$$\mathcal{P}(C) = \{A, B\}$$

$$\mathcal{P}(A) = \emptyset$$



Expectation maximization: E-step (grouping)

Grouping Q around θ_c:

$$Q(\theta_c) = \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X}|\mathbf{C}^{(s)},\boldsymbol{\Psi}} \left[\log P \left(\mathbf{X}, \mathbf{C}^{(s)} \mid \boldsymbol{\Psi} \right) \right]$$

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Grouping Q around θ_c:

$$Q(\theta_c) = \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X}|\mathbf{C}^{(s)},\boldsymbol{\Psi}} \left[\log P \left(\mathbf{X}, \mathbf{C}^{(s)} \mid \boldsymbol{\Psi} \right) \right]$$

$$= \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X}|\mathbf{C}^{(s)},\boldsymbol{\Psi}} \Bigg[\sum_{c_i \in s} \Bigg(\mathcal{I} \left(X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \right) \log(\theta_c) + \mathcal{I} \left(X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \right) \log(1 - \theta_c) \Bigg) + \mathcal{Z} \Bigg]$$

Expectation maximization: E-step (grouping)

Grouping Q around θ_c:

$$Q(\theta_c) = \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X}|\mathbf{C}^{(s)},\boldsymbol{\Psi}} \left[\log P\left(\mathbf{X}, \mathbf{C}^{(s)} \mid \boldsymbol{\Psi}\right) \right]$$

$$= \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X}|\mathbf{C}^{(s)},\boldsymbol{\Psi}} \left[\sum_{c_i \in s} \left(\mathcal{I}\left(X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p}\right) \log(\theta_c) + \right.\right.$$

$$\left.\left. \mathcal{I}\left(X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p}\right) \log(1 - \theta_c) \right) + \mathcal{Z} \right]$$

$$= \sum_{s \in \mathcal{S}} \sum_{c_i \in s} \left(P\left(X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right) \log(\theta_c) + \right.$$

$$\left. P\left(X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right) \log(1 - \theta_c) \right) + \mathcal{Z},$$

Expectation maximization: M-step

$$ESS(x) = \sum_{s \in \mathcal{S}} \sum_{c_i \in s} P\left(X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right),$$

Expectation maximization: M-step

$$ESS(x) = \sum_{s \in \mathcal{S}} \sum_{c_i \in s} P\left(X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right),$$

$$\frac{\partial Q(\theta_c)}{\partial \theta_c}$$

$$= \sum_{s \in \mathcal{S}} \sum_{c_i \in s} \left(\frac{P\left(X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}{\theta_c} - \frac{P\left(X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}{1 - \theta_c} \right)$$

$$= 0.$$

Expectation maximization: M-step

$$\theta_c^{(t+1)} = \frac{\sum_{s \in \mathcal{S}} \sum_{c_i \in s} P\left(X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}{\sum_{s \in \mathcal{S}} \sum_{c_i \in s} \sum_{x=0}^{x=1} P\left(X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}$$

Expectation maximization: M-step

$$\theta_c^{(t+1)} = \frac{\sum_{s \in \mathcal{S}} \sum_{c_i \in s} P\left(X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}{\sum_{s \in \mathcal{S}} \sum_{c_i \in s} \sum_{x=0}^{x=1} P\left(X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}$$

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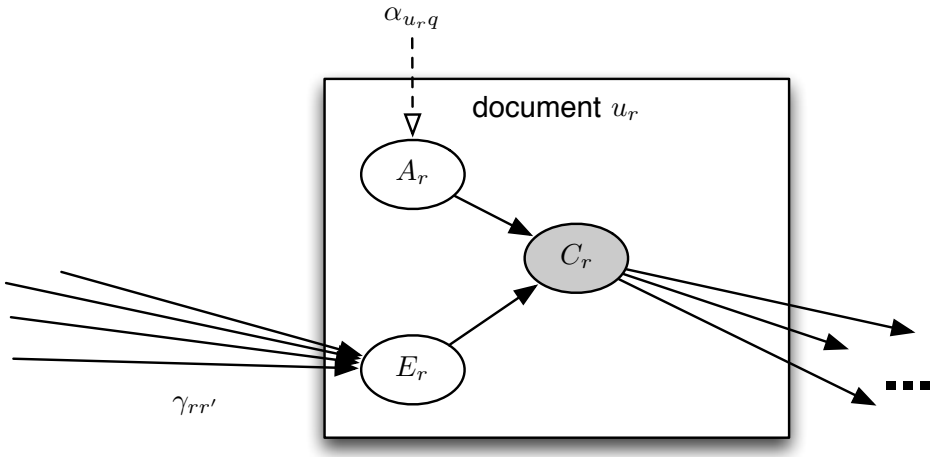
$$= \frac{\sum_{s \in \mathcal{S}} \sum_{c_i \in s} P\left(X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}{\sum_{s \in \mathcal{S}} \sum_{c_i \in s} P\left(\mathcal{P}(X_{c_i}^{(s)}) = \mathbf{p} \mid \mathbf{C}^{(s)}, \boldsymbol{\Psi}\right)}$$

$$= \frac{ESS^{(t)}(1)}{ESS^{(t)}(1) + ESS^{(t)}(0)}$$

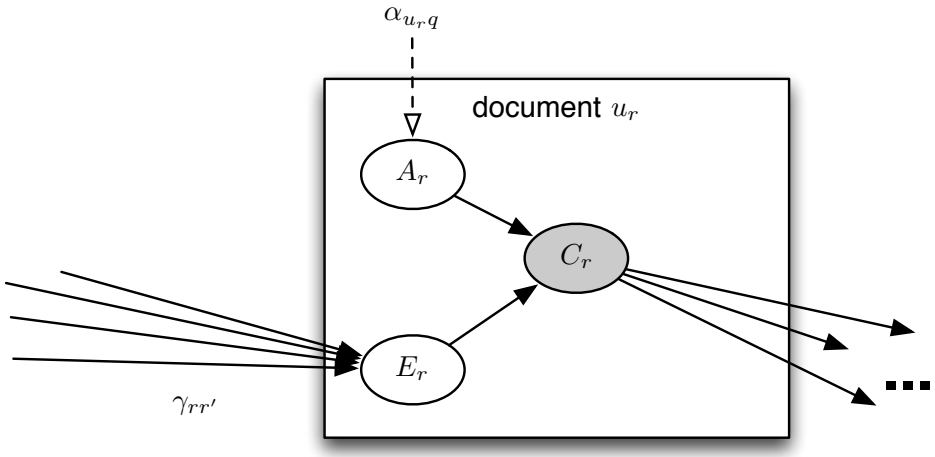
EM for user browsing model



$$P(A_u = 1) = \alpha_{uq}$$

$$P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'}$$

EM for user browsing model



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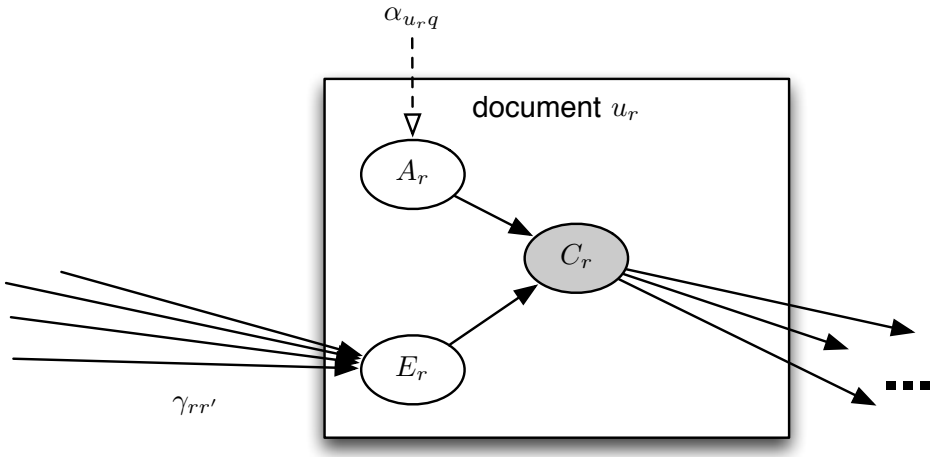
EM for user browsing model



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$$\mathcal{P}(A_u) = \emptyset$$

$$\mathcal{P}(E_r) = \{C_1, \ldots, C_{r-1}\}$$

EM for user browsing model: Attractiveness

$$P(A_u = 1) = \alpha_{uq}$$

EM for user browsing model: Attractiveness

$$P(A_u = 1) = \alpha_{uq}$$

$$P(A_u = 1, \mathcal{P}(A_u) = \mathbf{p} \mid \mathbf{C}) = P(A_u = 1 \mid \mathbf{C})$$
$$P(\mathcal{P}(A_u) = \mathbf{p} \mid \mathbf{C}) = 1$$

EM for user browsing model: Attractiveness

$$P(A_u = 1) = \alpha_{uq}$$

$$P(A_u = 1, \mathcal{P}(A_u) = \mathbf{p} \mid \mathbf{C}) = P(A_u = 1 \mid \mathbf{C})$$
$$P(\mathcal{P}(A_u) = \mathbf{p} \mid \mathbf{C}) = 1$$

$$\alpha_{uq}^{(t+1)} = \frac{\sum_{s \in \mathcal{S}_{uq}} P(A_u = 1 \mid \mathbf{C})}{\sum_{s \in \mathcal{S}_{uq}} 1} = \frac{1}{|\mathcal{S}_{uq}|} \sum_{s \in \mathcal{S}_{uq}} P(A_u = 1 \mid \mathbf{C})$$

EM for user browsing model: Attractiveness

$$P(A_u = 1 \mid \mathbf{C}) = P(A_u = 1 \mid C_u)$$

EM for user browsing model: Attractiveness

$$
\begin{aligned}
P(A_u = 1 \mid \mathbf{C}) &= P(A_u = 1 \mid C_u) \\
&= \mathcal{I}(C_u = 1) P(A_u = 1 \mid C_u = 1) + \\
&\quad \mathcal{I}(C_u = 0) P(A_u = 1 \mid C_u = 0)
\end{aligned}
$$

EM for user browsing model: Attractiveness

$$
\begin{aligned}
P(A_u = 1 \mid \mathbf{C}) &= P(A_u = 1 \mid C_u) \\
&= \mathcal{I}(C_u = 1) P(A_u = 1 \mid C_u = 1) + \\
&\quad \mathcal{I}(C_u = 0) P(A_u = 1 \mid C_u = 0) \\
&= c_u + (1 - c_u) \frac{P(C_u = 0 \mid A_u = 1) P(A_u = 1)}{P(C_u = 0)}
\end{aligned}
$$

EM for user browsing model: Attractiveness

$$
\begin{aligned}
P(A_u = 1 \mid \mathbf{C}) &= P(A_u = 1 \mid C_u) \\
&= \mathcal{I}(C_u = 1) P(A_u = 1 \mid C_u = 1) + \\
&\quad \mathcal{I}(C_u = 0) P(A_u = 1 \mid C_u = 0) \\
&= c_u + (1 - c_u) \frac{P(C_u = 0 \mid A_u = 1) P(A_u = 1)}{P(C_u = 0)} \\
&= c_u + (1 - c_u) \frac{(1 - \gamma_{rr'}) \alpha_{uq}}{1 - \gamma_{rr'} \alpha_{uq}}
\end{aligned}
$$

EM for user browsing model: Attractiveness

$$\alpha_{uq}^{(t+1)} = \frac{1}{|\mathcal{S}_{uq}|} \sum_{s \in \mathcal{S}_{uq}} \left(c_u^{(s)} + (1 - c_u^{(s)}) \frac{(1 - \gamma_{rr'}^{(t)}) \alpha_{uq}^{(t)}}{1 - \gamma_{rr'}^{(t)} \alpha_{uq}^{(t)}} \right)$$

EM for user browsing model: Examination

$$P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'}$$

EM for user browsing model: Examination

$$P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'}$$

$$\mathcal{P}(E_r) = \{C_1, \ldots, C_{r-1}\}$$
$$\mathbf{p} = [c_1, \ldots, c_{r'-1}, 1, 0, \ldots, 0]$$

EM for user browsing model: Examination

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$$P(E_r = x, \mathcal{P}(E_r) = \mathbf{p} \mid \mathbf{C}) = P(E_r = x \mid \mathbf{C})$$
$$P(\mathcal{P}(E_r) = \mathbf{p} \mid \mathbf{C}) = 1$$

EM for user browsing model: Examination

$$\gamma_{rr'}^{(t+1)} = \frac{\sum_{s \in \mathcal{S}_{rr'}} P(E_r = 1 \mid \mathbf{C})}{\sum_{s \in \mathcal{S}_{rr'}} 1} = \frac{1}{|\mathcal{S}_{rr'}|} \sum_{s \in \mathcal{S}_{rr'}} P(E_r = 1 \mid \mathbf{C})$$

EM for user browsing model: Examination

$$\gamma_{rr'}^{(t+1)} = \frac{\sum_{s \in \mathcal{S}_{rr'}} P(E_r = 1 \mid \mathbf{C})}{\sum_{s \in \mathcal{S}_{rr'}} 1} = \frac{1}{|\mathcal{S}_{rr'}|} \sum_{s \in \mathcal{S}_{rr'}} P(E_r = 1 \mid \mathbf{C})$$

$$P(E_r = 1 \mid \mathbf{C}) = c_u + (1 - c_u) \frac{\gamma_{rr'}(1 - \alpha_{uq})}{1 - \gamma_{rr'} \alpha_{uq}}$$

EM for user browsing model: Examination

$$\gamma_{rr'}^{(t+1)} = \frac{1}{|\mathcal{S}_{rr'}|} \sum_{s \in \mathcal{S}_{rr'}} \left(c_u^{(s)} + (1 - c_u^{(s)}) \frac{\gamma_{rr'}^{(t)} (1 - \alpha_{uq}^{(t)})}{1 - \gamma_{rr'}^{(t)} \alpha_{uq}^{(t)}} \right)$$

EM for user browsing model

$$\alpha_{uq}^{(t+1)} = \frac{1}{|\mathcal{S}_{uq}|} \sum_{s \in \mathcal{S}_{uq}} \left(c_u^{(s)} + (1 - c_u^{(s)}) \frac{(1 - \gamma_{rr'}^{(t)}) \alpha_{uq}^{(t)}}{1 - \gamma_{rr'}^{(t)} \alpha_{uq}^{(t)}} \right)$$

$$\gamma_{rr'}^{(t+1)} = \frac{1}{|\mathcal{S}_{rr'}|} \sum_{s \in \mathcal{S}_{rr'}} \left(c_u^{(s)} + (1 - c_u^{(s)}) \frac{\gamma_{rr'}^{(t)} (1 - \alpha_{uq}^{(t)})}{1 - \gamma_{rr'}^{(t)} \alpha_{uq}^{(t)}} \right)$$

Alternative estimation methods

- Bayesian inference

Alternative estimation methods

- Bayesian inference
- Probit link

Alternative estimation methods

- Bayesian inference
- Probit link
- Matrix factorization

Morning block 1 – Outline

Morning block 1 – Recap

- Basic click models as probabilistic graphical models
- MLE: Precise parameter estimation method for simple cases
- EM: Approximate parameter estimation for complex cases

Morning block 1 – Recap

- Basic click models as probabilistic graphical models
- MLE: Precise parameter estimation method for simple cases
- EM: Approximate parameter estimation for complex cases
- After the break
  - Demo
  - Evaluation
  - Data and tools
  - Experimental results

Acknowledgments

All content represents the opinion of the authors which is not necessarily shared or endorsed by their respective employers and/or sponsors.