An Introduction to Click Models for Web Search
(Morning block 1)

Aleksandr Chuklin§,¶
a.chuklin@uva.nl

Ilya Markov§
i.markov@uva.nl

Maarten de Rijke§
derijke@uva.nl

§University of Amsterdam
¶Google Switzerland

SIGIR 2015 Tutorial
Who are we?

Aleksandr
Software engineer
at Google
Who are we?

Aleksandr
Software engineer
at Google

Ilya
Postdoc at
U. Amsterdam
Who are we?

Aleksandr
Software engineer at Google

Ilya
Postdoc at U. Amsterdam

Maarten
Professor at U. Amsterdam
Aims

- **Describe existing click models** in a unified way, so that different models can easily be related to each other.
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- Provide **ready-to-use formulas and implementations** of existing click models and detail parameter estimation procedures to facilitate the development of new ones
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- Summarize current efforts on **click model evaluation** – evaluation approaches, datasets and software packages
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- **Compare** commonly used click models
- Provide **ready-to-use formulas and implementations** of existing click models and detail parameter estimation procedures to facilitate the development of new ones
- Summarize current efforts on **click model evaluation** – evaluation approaches, datasets and software packages
- Provide an overview of click model **applications and directions** for future development of click models
Structure of the tutorial

- Two parts, with two blocks each
  - An Introduction to Click Models for Web Search
  - Advanced Click Models and their applications to IR
Structure of the tutorial

- Two parts, with two blocks each
  - An Introduction to Click Models for Web Search
  - Advanced Click Models and their applications to IR

- Part I (this morning)
  - Introduction
  - Basic click models
  - Inference for click models
  - Break
  - Demo
  - Evaluation
  - Data and tools
  - Results on the basic click models
  - Recap
Materials

Complete draft of the book on which the tutorial is based:

Materials

- Complete draft of the book on which the tutorial is based:

- Copy of the slides
  - Parts I and II

Copy of the slides and code and data samples to follow live demos. See http://clickmodels.weebly.com for updates and additional materials.
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# Morning block 1 – Outline

1. **Introduction**
2. **Motivation**
3. **Basic Click Models**
4. **Click Probabilities**
5. **Parameter Estimation**
6. **Recap**
What is a click model?

Santiago, Chile

Wikipedia, the free encyclopedia

Santiago, officially Santiago de Chile, is the federal capital of Chile, and the center of its largest metropolitan area, known as "Greater Santiago."

Santiago, Chile - New World Encyclopedia

Santiago, officially Santiago de Chile, is the federal capital of Chile, and the center of its largest metropolitan area, known as "Greater Santiago."

Santiago, Chile - Lonely Planet

Chile & Easter Island - Santiago (Chapter). Santiago has always had its measured charms — fine dining, perfectly landscaped gardens, a famous seafood market...

Santiago de Chile travel guide - Wikitravel

Santiago de Chile is a huge city with several district articles containing sightseeing, restaurant, nightlife and accommodation listings — have a look at each of them. Santiago de Chile, usually shortened to Santiago, is the capital and economic cent...

Santiago, Chile

image_shield = Coat of arms of Santiago, Chile.svg. mapsizex = 200px map_caption = Location of Santiago commune in Greater Santiago subdivision_type = Region...
Notation

<table>
<thead>
<tr>
<th>Expression</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>A document</td>
</tr>
<tr>
<td>$q$</td>
<td>A user’s query</td>
</tr>
<tr>
<td>$r$</td>
<td>The rank of a document</td>
</tr>
<tr>
<td>$u_r$</td>
<td>A document at rank $r$</td>
</tr>
<tr>
<td>$r_u$</td>
<td>The rank of a document $u$</td>
</tr>
</tbody>
</table>
### Notation

<table>
<thead>
<tr>
<th>Expression</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>A placeholder for any concept associated with a SERP (e.g., query-document pair, rank, etc.)</td>
</tr>
<tr>
<td>$S$</td>
<td>A set of user search sessions</td>
</tr>
<tr>
<td>$S_c$</td>
<td>A set of user search sessions containing a concept $c$</td>
</tr>
<tr>
<td>$X_c$</td>
<td>An event $X$ applied to a concept $c$</td>
</tr>
<tr>
<td>$x_c$</td>
<td>The value that a random variable $X$ takes, when applied to a concept $c$</td>
</tr>
<tr>
<td>$x_c(s)$</td>
<td>The value that a random variable $X$ takes, when applied to a concept $c$ in a particular session $s$</td>
</tr>
</tbody>
</table>
Morning block 1 – Outline

1. Introduction
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6. Recap
Why click models?
Why click models?

- Understand users
Why click models?

- Understand users
- Simulate users
Why click models?

- Understand users
- Simulate users
- Evaluate search
Why click models?

- Understand users
- Simulate users
- Evaluate search
- Improve search
Morning block 1 – Outline

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Basic click models

- Random click model
- CTR models
- Position-based model
- Cascade model
- Dependent click model
- Dynamic Bayesian network model
- User browsing model
Random click model
Random click model

\[ P(C_u = 1) = \rho \]
CTR models

Rank-based CTR:

\[ P(C_r = 1) = \rho_r \]

Document-based CTR:

\[ P(C_u = 1) = \rho_{uq} \]
Position-based model

$$\gamma_{ru}$$

$$\alpha_{uq}$$

$$E_u$$

$$A_u$$

$$C_u$$

document $u$
Position-based model

\[ P(C_u = 1) = P(E_u = 1) \cdot P(A_u = 1) \]

\[ P(A_u = 1) = \alpha_{uq} \]

\[ P(E_u = 1) = \gamma_{ru} \]
Cascade model

\[ \alpha_{u_{r-1}, q} \]

\[ \alpha_{u, q} \]

Diagram showing the Cascade model with nodes labeled as follows:
- Document \( u_{r-1} \)
- Click \( C_{r-1} \)
- Ranking \( A_{r-1} \)
- Expected rank \( E_{r-1} \)
- Document \( u_r \)
- Click \( C_r \)
- Ranking \( A_r \)
- Expected rank \( E_r \)
Cascade model

\[ E_r = 1 \text{ and } A_r = 1 \iff C_r = 1 \]

\[ P(A_r = 1) = \alpha_{ur} q \]

\[ P(E_1 = 1) = 1 \]

\[ P(E_r = 1 \mid E_{r-1} = 0) = 0 \]

\[ P(E_r = 1 \mid C_{r-1} = 1) = 0 \]

\[ P(E_r = 1 \mid E_{r-1} = 1, C_{r-1} = 0) = 1 \]
Dependent click model
Dependent click model

\[
P(E_r = 1 \mid S_{r-1} = 1) = 0
\]

\[
P(E_r = 1 \mid E_{r-1} = 1, S_{r-1} = 0) = 1
\]

\[
P(S_r = 1 \mid C_r = 0) = 0
\]

\[
P(S_r = 1 \mid C_r = 1) = 1 - \lambda_r
\]
Dynamic Bayesian network model

\[ \alpha_{ur-1q} \quad \sigma_{ur-1q} \]

\[ A_{r-1} \quad S_{r-1} \]

\[ \gamma \]

\[ E_{r-1} \quad C_{r-1} \]

\[ \alpha_{urq} \quad \sigma_{urq} \]

\[ A_r \quad S_r \]

\[ \gamma \]

\[ E_r \quad C_r \]

\[ \text{document } u_{r-1} \]

\[ \text{document } u_r \]
Dynamic Bayesian network model

\[
P(E_r = 1 \mid S_{r-1} = 1) = 0
\]

\[
P(E_r = 1 \mid E_{r-1} = 1, S_{r-1} = 0) = \gamma
\]

\[
P(S_r = 1 \mid C_r = 1) = \sigma_{u_r}q
\]
User browsing model

\[ \alpha_{u_r q} \]

\[ \gamma_{r r'} \]

document \( u_r \)

\[ A_r \]

\[ C_r \]

\[ E_r \]

\[ \ldots \]
User browsing model

\[
P(E_r = 1 | C_{r'}, C_{r'+1}, \ldots, C_{r-1} = 0) = \gamma_{rr'}
\]
## Basic click models summary

<table>
<thead>
<tr>
<th>Model</th>
<th>$P(A_u = 1)$</th>
<th>$P(S_u = 1 \mid C_u = 1)$</th>
<th>$P(E_u = 1 \mid E &lt; r_u, S &lt; r_u, C &lt; r_u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCM</td>
<td>constant ($\rho$)</td>
<td>N/A</td>
<td>constant (1)</td>
</tr>
<tr>
<td>RCTR</td>
<td>constant (1)</td>
<td>N/A</td>
<td>rank ($\rho_r$)</td>
</tr>
<tr>
<td>DCTR</td>
<td>query-document ($\rho_{uq}$)</td>
<td>N/A</td>
<td>constant (1)</td>
</tr>
<tr>
<td>PBM</td>
<td>query-doc ($\alpha_{uq}$)</td>
<td>N/A</td>
<td>rank ($\gamma_r$)</td>
</tr>
<tr>
<td>CM</td>
<td>query-doc ($\alpha_{uq}$)</td>
<td>constant (1)</td>
<td>constant (1 or 0)</td>
</tr>
<tr>
<td>DCM</td>
<td>query-doc ($\alpha_{uq}$)</td>
<td>rank ($1 - \lambda_r$)</td>
<td>constant (1 or 0)</td>
</tr>
<tr>
<td>DBN</td>
<td>query-doc ($\alpha_{uq}$)</td>
<td>query-doc ($\sigma_{uq}$)</td>
<td>constant ($\gamma$)</td>
</tr>
<tr>
<td>UBM</td>
<td>query-doc ($\alpha_{uq}$)</td>
<td>N/A</td>
<td>other ($\gamma_{rr'}$)</td>
</tr>
</tbody>
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## Click probabilities

<table>
<thead>
<tr>
<th>Full probability:</th>
<th>$P(C_r = 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional probability:</td>
<td>$P(C_r = 1</td>
</tr>
</tbody>
</table>
Click probabilities

- Full probability:

\[ P(C_r = 1) \]
Click probabilities

- Full probability:

\[ P(C_r = 1) \]

- Conditional probability:

\[ P(C_r = 1 \mid C_1, \ldots, C_{r-1}) \]
Full click probability

\[ P(C_u = 1) = P(C_u = 1 \mid E_{ru} = 1) \cdot P(E_{ru} = 1) = \alpha_{uq} \epsilon_{ru} \]
Full click probability

\[
P(C_u = 1) = P(C_u = 1 \mid E_{ru} = 1) \cdot P(E_{ru} = 1) = \alpha_{uq} \epsilon_{r_u}
\]

\[
\epsilon_{r+1} = P(E_{r+1} = 1)
\]
Full click probability

\[
P(C_u = 1) = P(C_u = 1 \mid E_{ru} = 1) \cdot P(E_{ru} = 1) = \alpha_{uq} \epsilon_{ru}
\]

\[
\epsilon_{r+1} = P(E_{r+1} = 1) = P(E_r = 1) \cdot P(E_{r+1} = 1 \mid E_r = 1)
\]
Full click probability

\[ P(C_u = 1) = P(C_u = 1 \mid E_{ru} = 1) \cdot P(E_{ru} = 1) = \alpha_{uq} \epsilon_{ru} \]

\[ \epsilon_{r+1} = P(E_{r+1} = 1) \]
\[ = P(E_r = 1) \cdot P(E_{r+1} = 1 \mid E_r = 1) \]
\[ = \epsilon_r \cdot (P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(C_r = 1 \mid E_r = 1) + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(C_r = 0 \mid E_r = 1)) \]
Full click probability

\[ P(C_u = 1) = P(C_u = 1 \mid E_{ru} = 1) \cdot P(E_{ru} = 1) = \alpha_{uq}\epsilon_{ru} \]

\[ \epsilon_{r+1} = P(E_{r+1} = 1) \\
= P(E_r = 1) \cdot P(E_{r+1} = 1 \mid E_r = 1) \\
= \epsilon_r \cdot (P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(C_r = 1 \mid E_r = 1) + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(C_r = 0 \mid E_r = 1)) \]

DBN: \[ \epsilon_{r+1} = \epsilon_r ((1 - \sigma_{uq}) \gamma \alpha_{uq} + \gamma (1 - \alpha_{uq})) \]
Conditional click probability

\[ P(C_u = 1 \mid C_{<r_u}) = P(C_u = 1 \mid E_{r_u} = 1, C_{<r_u}) \cdot P(E_{r_u} = 1 \mid C_{<r_u}) = \alpha_{uq}\epsilon_{r_u} \]
Conditional click probability

\[ P(C_u = 1 \mid C_{<r_u}) = P(C_u = 1 \mid E_{ru} = 1, C_{<r_u}) \cdot P(E_{ru} = 1 \mid C_{<r_u}) \]

\[ = \alpha_{uq} \epsilon_{ru} \]

\[ \epsilon_{r+1} = P(E_{r+1} = 1 \mid C_{<r+1}) \]
Conditional click probability

\[ P(C_u = 1 \mid C_{<r}) = P(C_u = 1 \mid E_{ru} = 1, C_{<r}) \cdot P(E_{ru} = 1 \mid C_{<r}) = \alpha_{uq} \epsilon_{ru} \]

\[ \epsilon_{r+1} = P(E_{r+1} = 1 \mid C_{<r+1}) = P(E_{r+1} = 1 \mid E_r = 1, C_{<r+1}) \cdot P(E_r = 1 \mid C_{<r+1}) \]
Conditional click probability

\[
P(C_u = 1 \mid C_{<u}) = P(C_u = 1 \mid E_{r_u} = 1, C_{<u}) \cdot P(E_{r_u} = 1 \mid C_{<u}) \\
= \alpha_u \epsilon_{r_u}
\]

\[
\epsilon_{r+1} = P(E_{r+1} = 1 \mid C_{<r+1}) \\
= P(E_{r+1} = 1 \mid E_r = 1, C_{<r+1}) \cdot P(E_r = 1 \mid C_{<r+1}) \\
= P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(E_r = 1 \mid C_r = 1, C_{<r}) \cdot c_r^{(s)} + \\
P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(E_r = 1 \mid C_r = 0, C_{<r}) \cdot (1 - c_r^{(s)})
\]
Conditional click probability

\[ P(C_u = 1 \mid C_{< r_u}) = P(C_u = 1 \mid E_{r_u} = 1, C_{< r_u}) \cdot P(E_{r_u} = 1 \mid C_{< r_u}) \]

\[ = \alpha_{uq} \epsilon_{r_u} \]

\[ \epsilon_{r+1} = P(E_{r+1} = 1 \mid C_{< r+1}) \]

\[ = P(E_{r+1} = 1 \mid E_r = 1, C_{< r+1}) \cdot P(E_r = 1 \mid C_{< r+1}) \]

\[ = P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(E_r = 1 \mid C_r = 1, C_{< r}) \cdot c_r^{(s)} + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(E_r = 1 \mid C_r = 0, C_{< r}) \cdot (1 - c_r^{(s)}) \]

\[ = P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot c_r^{(s)} + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot \frac{\epsilon_r(1 - \alpha_{uq})}{1 - \alpha_{uq} \epsilon_r} (1 - c_r^{(s)}) \]
Conditional click probability

\[ P(C_u = 1 \mid C_{<r_u}) = P(C_u = 1 \mid E_{r_u} = 1, C_{<r_u}) \cdot P(E_{r_u} = 1 \mid C_{<r_u}) \]

\[ = \alpha_{uq}\epsilon_{r_u} \]

\[ \epsilon_{r+1} = P(E_{r+1} = 1 \mid C_{<r+1}) \]
\[ = P(E_{r+1} = 1 \mid E_r = 1, C_{<r+1}) \cdot P(E_r = 1 \mid C_{<r+1}) \]
\[ = P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(E_r = 1 \mid C_r = 1, C_{<r}) \cdot c_r^{(s)} + \]
\[ P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(E_r = 1 \mid C_r = 0, C_{<r}) \cdot (1 - c_r^{(s)}) \]
\[ = P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot c_r^{(s)} + \]
\[ P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot \frac{\epsilon_r(1 - \alpha_{uq})}{1 - \alpha_{uq}\epsilon_r} \cdot c_r^{(s)} \]

**DCM:** \[ \epsilon_{r+1} = \lambda_r c_r^{(s)} + \frac{(1 - \alpha_{uq})\epsilon_r}{1 - \alpha_{uq}\epsilon_r} \cdot c_r^{(s)} \]
<table>
<thead>
<tr>
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<th></th>
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<tbody>
<tr>
<td>1 Introduction</td>
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<tr>
<td>6 Recap</td>
<td></td>
</tr>
</tbody>
</table>
Click model

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</tr>
</thead>
</table>

A click model is a set of events/random variables, a set of dependencies between these events, and a correspondence between the model's parameters and features of a query and results.
Click model

- Set of events/random variables
- Set of dependencies between these events
- Correspondence between the model’s parameters and features of a query and results
Click model

- Set of events/random variables
- Set of dependencies between these events
- Correspondence between the model’s parameters and features of a query and results
Parameter estimation

- Maximum likelihood estimation
- Expectation maximization
- Alternative estimation methods
MLE for random click model

\[ P(C_u = 1) = \rho \]
MLE for random click model

\[ P(C_u = 1) = \rho \]

\[ \mathcal{L} = \prod_{s \in S} \prod_{u \in s} \rho^{c_u^{(s)}} (1 - \rho)^{1 - c_u^{(s)}} \]
MLE for random click model

\[ P(C_u = 1) = \rho \]

\[
\mathcal{L} = \prod_{s \in S} \prod_{u \in s} \rho^{c_u^{(s)}} (1 - \rho)^{1 - c_u^{(s)}}
\]

\[
\mathcal{L} \mathcal{L} = \sum_{s \in S} \sum_{u \in s} \left( c_u^{(s)} \log(\rho) + (1 - c_u^{(s)}) \log(1 - \rho) \right)
\]
MLE for random click model

\[ P(C_u = 1) = \rho \]

\[ \mathcal{L} = \prod_{s \in S} \prod_{u \in s} \rho^{c_u^{(s)}} (1 - \rho)^{1-c_u^{(s)}} \]

\[ \mathcal{L} \mathcal{L} = \sum_{s \in S} \sum_{u \in s} \left( c_u^{(s)} \log(\rho) + (1 - c_u^{(s)}) \log(1 - \rho) \right) \]

\[ \rho = \frac{\sum_{s \in S} \sum_{u \in s} c_u^{(s)}}{\sum_{s \in S} |s|} \]
MLE for dependent click model

\[ P(A_u = 1) = \alpha_u q \]

\[ P(E_r = 1 \mid E_{r-1} = 1, S_{r-1}) = 1 - S_{r-1} \]

\[ P(S_r = 1 \mid C_r = 0) = 0 \]

\[ P(S_r = 1 \mid C_r = 1) = 1 - \lambda_r \]

\[ C_u = 1 \iff E_{ru} = 1, A_u = 1 \]
MLE for dependent click model

\[ P(A_u = 1) = \alpha_{uq} \]

\[ P(E_r = 1 \mid E_{r-1} = 1, S_{r-1}) = 1 - S_{r-1} \]

\[ P(S_r = 1 \mid C_r = 0) = 0 \]

\[ P(S_r = 1 \mid C_r = 1) = 1 - \lambda_r \]

\[ C_u = 1 \iff E_{ru} = 1, A_u = 1 \]

\[ S_r = 1 \iff r = l, \]

where \( l \) is the last-clicked rank
MLE for dependent click model: Attractiveness

\[ P(A_u = 1) = \alpha_{uq} \]

\[ \forall r \leq l : A_{ur} = 1 \iff C_{ur} = 1 \]
MLE for dependent click model: Attractiveness

\[ P(A_u = 1) = \alpha_{uq} \]
\[ \forall r \leq l: \quad A_{ur} = 1 \iff C_{ur} = 1 \]

\[
\mathcal{L}(\alpha_{uq}) = \prod_{s \in S_{uq}} \alpha_{uq}^{I(A_u^{(s)}=1)} (1 - \alpha_{uq})^{1-I(A_u^{(s)}=1)}
\]

\[
\mathcal{L}\mathcal{L}(\alpha_{uq}) = \sum_{s \in S_{uq}} (I(\ldots) \log(\alpha_{uq}) + (1 - I(\ldots)) \log(1 - \alpha_{uq}))
\]
MLE for dependent click model: Attractiveness

\[ P(A_u = 1) = \alpha_{uq} \]

\[ \forall r \leq l : A_{ur} = 1 \iff C_{ur} = 1 \]

\[ \mathcal{L}(\alpha_{uq}) = \prod_{s \in S_{uq}} \alpha_{uq}^{I(A_u^{(s)} = 1)} (1 - \alpha_{uq})^{1 - I(A_u^{(s)} = 1)} \]

\[ \mathcal{L}\mathcal{L}(\alpha_{uq}) = \sum_{s \in S_{uq}} (I(\ldots) \log(\alpha_{uq}) + (1 - I(\ldots)) \log(1 - \alpha_{uq})) \]

\[ \alpha_{uq} = \frac{\sum_{s \in S_{uq}} I(A_u^{(s)} = 1)}{|S_{uq}|} = \frac{\sum_{s \in S_{uq}} I(C_u^{(s)} = 1)}{|S_{uq}|} = \frac{\sum_{s \in S_{uq}} c_u^{(s)}}{|S_{uq}|} \]
MLE for dependent click model: Continuation

\[ P(S_r = 0) = \lambda_r \]

\[ S_r = 1 \iff C_r = 1, r = l \]
MLE for dependent click model: Continuation

\[ P(S_r = 0) = \lambda_r \]

\[ S_r = 1 \iff C_r = 1, \ r = l \]

\[ \mathcal{L}(\lambda_r) = \prod_{s \in S_r} \lambda_r^{I(S_r^{(s)}=0)}(1 - \lambda_r)^{1-I(S_r^{(s)}=1)} \]
MLE for dependent click model: Continuation

\[ P(S_r = 0) = \lambda_r \]

\[ S_r = 1 \iff C_r = 1, \ r = l \]

\[ \mathcal{L}(\lambda_r) = \prod_{s \in S_r} \lambda_r^{\mathcal{I}(S_r^{(s)}=0)} (1 - \lambda_r)^{1 - \mathcal{I}(S_r^{(s)}=1)} \]

\[ \lambda_r = \frac{\sum_{s \in S_r} \mathcal{I}(S_r^{(s)} = 0)}{|S_r|} \]
MLE for dependent click model: Continuation

\[ P(S_r = 0) = \lambda_r \]

\[ S_r = 1 \iff C_r = 1, \ r = l \]

\[ \mathcal{L}(\lambda_r) = \prod_{s \in S_r} \lambda_r^{I(S_r^{(s)}=0)} (1 - \lambda_r)^{1-I(S_r^{(s)}=1)} \]

\[ \lambda_r = \frac{\sum_{s \in S_r} \mathcal{I}(S_r^{(s)} = 0)}{|S_r|} = \frac{\sum_{s \in S_r} (1 - \mathcal{I}(r = l))}{|S_r|} \]
MLE for dependent click model: Continuation

\[ P(S_r = 0) = \lambda_r \]

\[ S_r = 1 \iff C_r = 1, r = l \]

\[ \mathcal{L}(\lambda_r) = \prod_{s \in S_r} \lambda_r^{I(S_r^{(s)}=0)} (1 - \lambda_r)^{1-I(S_r^{(s)}=1)} \]

\[ \lambda_r = \frac{\sum_{s \in S_r} I(S_r^{(s)} = 0)}{|S_r|} = \frac{\sum_{s \in S_r} (1 - I(r = l))}{|S_r|} = \frac{\sum_{s \in S_r} I(r \neq l)}{|S_r|} \]
MLE for dependent click model

\[
\alpha_{uq} = \frac{1}{|S_{uq}|} \sum_{s \in S_{uq}} c_u^{(s)}
\]

\[
\lambda_r = \frac{1}{|S_r|} \sum_{s \in S_r} \mathbb{I}(r \neq l)
\]
Expectation maximization

\[
\mathcal{L} = \sum_{s \in S} \log \left( \sum_{X} P \left( X, C^{(s)} | \Psi \right) \right)
\]
Expectation maximization

\[ \mathcal{L} = \sum_{s \in S} \log \left( \sum_X P \left( X, C^{(s)} | \psi \right) \right) \]

\[ Q = \sum_{s \in S} \mathbb{E}_{X|C^{(s)}, \psi} \left[ \log P \left( X, C^{(s)} | \psi \right) \right] \]
Expectation maximization: E-step (grouping)

Each node $X$ depends only on its parents $\mathcal{P}(X)$
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$\mathcal{P}(C) = \{A, B\}$
Each node $X$ depends only on its parents $\mathcal{P}(X)$

$$\mathcal{P}(C) = \{A, B\}$$

$$\mathcal{P}(A) = \emptyset$$
Expectation maximization: E-step (grouping)

Grouping $Q$ around $\theta_c$:

$$Q(\theta_c) = \sum_{s \in S} \mathbb{E}_{X | C^{(s)}, \Psi} \left[ \log P \left( X, C^{(s)} | \Psi \right) \right]$$
Expectation maximization: E-step (grouping)

Grouping $Q$ around $\theta_c$:

$$Q(\theta_c) = \sum_{s \in S} \mathbb{E}_{X|C^{(s)}, \psi} \left[ \log P \left( X, C^{(s)} \mid \psi \right) \right]$$

$$= \sum_{s \in S} \mathbb{E}_{X|C^{(s)}, \psi} \left[ \sum_{c_i \in s} \left( \mathcal{I} \left( X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = p \right) \log(\theta_c) + \mathcal{I} \left( X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = p \right) \log(1 - \theta_c) \right) + \mathcal{Z} \right]$$
Expectation maximization: E-step (grouping)

Grouping $Q$ around $\theta_c$: 

$$
Q(\theta_c) = \sum_{s \in S} \mathbb{E}_{X \mid C^{(s)}, \Psi} \left[ \log P \left( X, C^{(s)} \mid \Psi \right) \right]
$$

$$
= \sum_{s \in S} \mathbb{E}_{X \mid C^{(s)}, \Psi} \left[ \sum_{c_i \in s} \left( I \left( X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = p \right) \log(\theta_c) +
\mathcal{I} \left( X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = p \right) \log(1 - \theta_c) \right) + \mathcal{Z} \right]
$$

$$
= \sum_{s \in S} \sum_{c_i \in s} \left( P \left( X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right) \log(\theta_c) +
P \left( X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right) \log(1 - \theta_c) \right) + \mathcal{Z},
$$
Expectation maximization: M-step

\[ \text{ESS}(x) = \sum_{s \in S} \sum_{c_i \in s} P \left( X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right), \]

\[ = \sum_{s \in S} \sum_{c_i \in s} P \left( X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right). \]
Expectation maximization: M-step

\[ ESS(x) = \sum_{s \in S} \sum_{c_i \in s} P \left( X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right), \]

\[
\frac{\partial Q(\theta_c)}{\partial \theta_c} = \sum_{s \in S} \sum_{c_i \in s} \left( \frac{P \left( X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right)}{\theta_c} - \frac{P \left( X_{c_i}^{(s)} = 0, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right)}{1 - \theta_c} \right)
\]

\[ = 0. \]
Expectation maximization: M-step

\[
\theta_{c}^{(t+1)} = \frac{\sum_{s \in S} \sum_{c_i \in s} P\left( X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right)}{\sum_{s \in S} \sum_{c_i \in s} \sum_{x=0}^{x=1} P\left( X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right)}
\]
Expectation maximization: M-step

\[
\theta_c^{(t+1)} = \frac{\sum_{s \in S} \sum_{c_i \in s} P \left( X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right)}{\sum_{s \in S} \sum_{c_i \in s} \sum_{x=0}^{1} P \left( X_{c_i}^{(s)} = x, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right)}
\]

\[
= \frac{\sum_{s \in S} \sum_{c_i \in s} P \left( X_{c_i}^{(s)} = 1, \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right)}{\sum_{s \in S} \sum_{c_i \in s} P \left( \mathcal{P}(X_{c_i}^{(s)}) = p \mid C^{(s)}, \Psi \right)}
\]
Expectation maximization: M-step

\[
\theta_{c}^{(t+1)} = \frac{\sum_{s \in S} \sum_{c_i \in s} P \left( X_{c_i}^{(s)} = 1, P(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right)}{\sum_{s \in S} \sum_{c_i \in s} \sum_{x=0}^{1} P \left( X_{c_i}^{(s)} = x, P(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right)}
\]

\[
= \frac{\sum_{s \in S} \sum_{c_i \in s} P \left( X_{c_i}^{(s)} = 1, P(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right)}{\sum_{s \in S} \sum_{c_i \in s} P \left( P(X_{c_i}^{(s)}) = p \mid C^{(s)}, \psi \right)}
\]

\[
= \frac{ESS^{(t)}(1)}{ESS^{(t)}(1) + ESS^{(t)}(0)}
\]
EM for user browsing model

\[ P(A_u = 1) = \alpha_{uq} \]

\[ P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'} \]
EM for user browsing model

\[ P(A_u = 1) = \alpha_{uq} \]

\[ P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'} \]

\[ \mathcal{P}(A_u) = \emptyset \]
EM for user browsing model

\[ P(A_u = 1) = \alpha_{uq} \]

\[ P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'} \]

\[ \mathcal{P}(A_u) = \emptyset \]

\[ \mathcal{P}(E_r) = \{C_1, \ldots, C_{r-1}\} \]
EM for user browsing model: Attractiveness

\[ P(A_u = 1) = \alpha_{uq} \]
EM for user browsing model: Attractiveness

\[ P(A_u = 1) = \alpha_{uq} \]

\[ P(A_u = 1, P(A_u) = p \mid C) = P(A_u = 1 \mid C) \]
\[ P(P(A_u) = p \mid C) = 1 \]
EM for user browsing model: Attractiveness

\[ P(A_u = 1) = \alpha_{uq} \]

\[ P(A_u = 1, \mathcal{P}(A_u) = p | C) = P(A_u = 1 | C) \]
\[ P(\mathcal{P}(A_u) = p | C) = 1 \]

\[ \alpha_{uq}^{(t+1)} = \frac{\sum_{s \in S_{uq}} P(A_u = 1 | C)}{\sum_{s \in S_{uq}} 1} = \frac{1}{|S_{uq}|} \sum_{s \in S_{uq}} P(A_u = 1 | C) \]
EM for user browsing model: Attractiveness

\[ P(A_u = 1 \mid C) = P(A_u = 1 \mid C_u) \]
EM for user browsing model: Attractiveness

\[ P(A_u = 1 \mid C) = P(A_u = 1 \mid C_u) \]

\[ = \mathcal{I}(C_u = 1)P(A_u = 1 \mid C_u = 1) + \]

\[ \mathcal{I}(C_u = 0)P(A_u = 1 \mid C_u = 0) \]
EM for user browsing model: Attractiveness

\[
P(A_u = 1 \mid C) = P(A_u = 1 \mid C_u)
\]
\[
= I(C_u = 1) P(A_u = 1 \mid C_u = 1) + I(C_u = 0) P(A_u = 1 \mid C_u = 0)
\]
\[
= c_u + (1 - c_u) \frac{P(C_u = 0 \mid A_u = 1) P(A_u = 1)}{P(C_u = 0)}
\]
EM for user browsing model: Attractiveness

\[
P(A_u = 1 \mid C) = P(A_u = 1 \mid C_u)
\]
\[
= \mathcal{I}(C_u = 1)P(A_u = 1 \mid C_u = 1) + \mathcal{I}(C_u = 0)P(A_u = 1 \mid C_u = 0)
\]
\[
= c_u + (1 - c_u) \frac{P(C_u = 0 \mid A_u = 1)P(A_u = 1)}{P(C_u = 0)}
\]
\[
= c_u + (1 - c_u) \frac{(1 - \gamma_{rr'})\alpha_{uq}}{1 - \gamma_{rr'}\alpha_{uq}}
\]
EM for user browsing model: Attractiveness

\[
\alpha_{uq}^{(t+1)} = \frac{1}{|S_{uq}|} \sum_{s \in S_{uq}} \left( c_u^{(s)} + (1 - c_u^{(s)}) \frac{(1 - \gamma_{rr'}^{(t)}) \alpha_{uq}^{(t)}}{1 - \gamma_{rr'}^{(t)} \alpha_{uq}^{(t)}} \right)
\]
EM for user browsing model: Examination

\[ P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'} \]
EM for user browsing model: Examination

\[ P(E_r = 1 \mid C_r' = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'} \]

\[ \mathcal{P}(E_r) = \{ C_1, \ldots, C_{r-1} \} \]

\[ \mathbf{p} = [c_1, \ldots, c_{r'-1}, 1, 0, \ldots, 0] \]
EM for user browsing model: Examination

\[ P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) = \gamma_{rr'} \]

\[ \mathcal{P}(E_r) = \{ C_1, \ldots, C_{r-1} \} \]
\[ p = [c_1, \ldots, c_{r'-1}, 1, 0, \ldots, 0] \]

\[ S_{rr'} = \{ s : c_{r'} = 1, c_{r'+1} = 0, \ldots, c_{r-1} = 0 \} \]
EM for user browsing model: Examination

\begin{align*}
P(E_r = 1 \mid C_{r'} = 1, C_{r'+1} = 0, \ldots, C_{r-1} = 0) &= \gamma_{rr'} \\
\mathcal{P}(E_r) &= \{C_1, \ldots, C_{r-1}\} \\
p &= [c_1, \ldots, c_{r'-1}, 1, 0, \ldots, 0] \\
S_{rr'} &= \{s : c_{r'} = 1, c_{r'+1} = 0, \ldots, c_{r-1} = 0\} \\
P(E_r = x, \mathcal{P}(E_r) = p \mid C) &= P(E_r = x \mid C) \\
P(\mathcal{P}(E_r) = p \mid C) &= 1
\end{align*}
EM for user browsing model: Examination

\[ \gamma_{rr'}^{(t+1)} = \frac{\sum_{s \in S_{rr'}} P(E_r = 1 \mid C)}{\sum_{s \in S_{rr'}} 1} = \frac{1}{|S_{rr'}|} \sum_{s \in S_{rr'}} P(E_r = 1 \mid C) \]
EM for user browsing model: Examination

\[
\gamma_{rr'}^{(t+1)} = \frac{\sum_{s \in S_{rr'}} P(E_r = 1 \mid C)}{\sum_{s \in S_{rr'}} 1} = \frac{1}{|S_{rr'}|} \sum_{s \in S_{rr'}} P(E_r = 1 \mid C)
\]

\[
P(E_r = 1 \mid C) = c_u + (1 - c_u) \frac{\gamma_{rr'}(1 - \alpha_{uq})}{1 - \gamma_{rr'} \alpha_{uq}}
\]
EM for user browsing model: Examination

\[
\gamma_{rr'}^{(t+1)} = \frac{1}{|S_{rr'}|} \sum_{s \in S_{rr'}} \left( c_u^{(s)} + (1 - c_u^{(s)}) \frac{\gamma_{rr'}^{(t)} (1 - \alpha_u^{(t)})}{1 - \gamma_{rr'}^{(t)} \alpha_u^{(t)}} \right)
\]
EM for user browsing model

\[
\alpha_{uq}^{(t+1)} = \frac{1}{|S_{uq}|} \sum_{s \in S_{uq}} \left( c_u^{(s)} + (1 - c_u^{(s)}) \frac{(1 - \gamma_{rr'}^{(t)}) \alpha_{uq}^{(t)}}{1 - \gamma_{rr'}^{(t)} \alpha_{uq}^{(t)}} \right)
\]

\[
\gamma_{rr'}^{(t+1)} = \frac{1}{|S_{rr'}|} \sum_{s \in S_{rr'}} \left( c_u^{(s)} + (1 - c_u^{(s)}) \frac{\gamma_{rr'}^{(t)} (1 - \alpha_{uq}^{(t)})}{1 - \gamma_{rr'}^{(t)} \alpha_{uq}^{(t)}} \right)
\]
Alternative estimation methods

- Bayesian inference
Alternative estimation methods

- Bayesian inference
- Probit link
Alternative estimation methods

- Bayesian inference
- Probit link
- Matrix factorization
Morning block 1 – Outline

1. Introduction
2. Motivation
3. Basic Click Models
4. Click Probabilities
5. Parameter Estimation
6. Recap
Basic click models as probabilistic graphical models
MLE: Precise parameter estimation method for simple cases
EM: Approximate parameter estimation for complex cases
Basic click models as probabilistic graphical models
MLE: Precise parameter estimation method for simple cases
EM: Approximate parameter estimation for complex cases

After the break
- Demo
- Evaluation
- Data and tools
- Experimental results
Acknowledgments

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